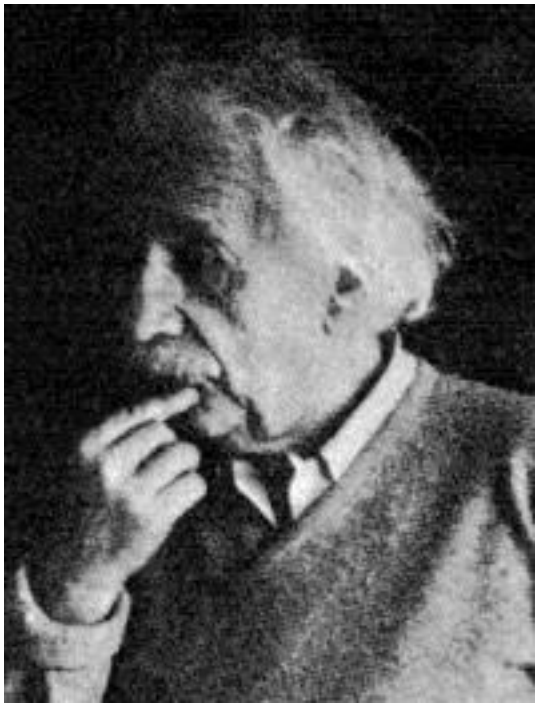
High Impact Practices for Reading and



Understanding Mathematics Applications

Hmmm, I wonder what would happen if . . .

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High Impact Practices for Reading and Understanding Mathematics Applications

The problem: How can we, as professional educators, help students improve their proficiency in learning from math textbooks and in solving word problems?

Factors to consider:

1. Over emphasis on covering content rather than teaching process, concepts, and connections within the material and outside of class.
   "Education is not the learning of facts, but the training of the mind to think," is a quotation attributed to Albert Einstein.

2. Student have difficulty processing material in math texts and show reluctance to work on solving word problems. According to Maxwell (1997), math "requires the ability to think abstractly and analytically, to reason logically and deductively, to translate words into symbols, to manipulate these symbols to solve word problems, and to integrate this information and apply it to practical situations" (p. 269).
   Often students have asked, "When are we going to use this stuff?" Application items, word problems, are one way of answering the question. Ironically, students hate math word problems and consider them extremely difficult. More than once, a student has shown the ability to solve the equation associated with an application item, but not the ability to translate the wording of the problem into the needed mathematical statements.

3. College level math requires students to use the higher order thinking skills (4–6) of Bloom's Taxonomy while they are more used to using the lower order skills (1–3). The key to operating in the higher level is the ability to think reflectively. In Bloom's original version, the six levels are: (1) Knowledge, (2) Comprehension, (3) Application, (4) Analysis, (5) Synthesis, (6) Evaluation. The Revised Taxonomy (2001) lists them as: (1) Remember, (2) Understand, (3) Apply, (4) Analyze, (5) Evaluate, (6) Create (Armstrong, n.d.).

Ways to improve student success with math and word problems:

Walker and Plata (2000) found confirmation in some research that the instructor is the factor most closely related to student success in mathematics. "Students . . . characterized the successful instructor as being sensitive to the mathematical and emotional needs of students, making the course fun and nonthreatening, and giving students the chance to succeed as soon as possible during the first class meeting." They conclude that the mathematics teacher must initiate "pragmatic teaching strategies" (30).

A. Incorporate reflection and reflective thinking skills in all teaching and learning activities.

What is needed for reflective thinking?

Rodgers (June 2002, p. 845) provides a distillation of John Dewey's four criteria for reflection:

1. Makes meaning: the learner moves from one experience to another gaining a deeper understanding of the relationships and connections to other experiences and ideas. The learner establishes the continuity needed for learning, ensuring individual and societal progress. Essentially a means to a moral ends.
2. Requires systematic, rigorous, disciplined, thinking rooted in scientific inquiry.
3. Happens as an interactive process within a community of others.
4. Requires attitudes valuing the growth of self and others personally and intellectually.

What is reflective thinking?

> ***Reflective thinking*** is a part of the critical thinking process referring specifically to the processes of analyzing and making judgments about what has happened. Dewey (1933) suggests that reflective thinking is an active, persistent, and careful consideration of a belief or supposed form of knowledge, of the grounds that support that knowledge, and the further conclusions to which that knowledge leads. Learners are aware of and control their learning by actively participating in reflective thinking – assessing what they know, what they need to know, and how they bridge that gap – during learning situations.
>
> In summary, ***critical thinking*** involves a wide range of thinking skills leading toward desirable outcomes and ***reflective thinking*** focuses on the process of making judgments about what has happened. However, reflective thinking is most important in prompting learning during complex problem-solving situations because it provides students with an opportunity to step back and think about how they actually solve problems and how a particular set of problem solving strategies is appropriated for achieving their goal. (University of Hawaii)

Why teach reflective thinking (Can help students develop a stronger growth mindset):

> Modern society is becoming more complex, information is becoming available and changing more rapidly prompting users to constantly rethink, switch directions, and change problem-solving strategies. Thus, it is increasingly important to prompt reflective thinking during learning to help learners develop strategies to apply new knowledge to the complex situations in their day-to-day activities. Reflective thinking helps learners develop higher-order thinking skills by prompting learners to a) relate new knowledge to prior understanding, b) think in both abstract and conceptual terms, c) apply specific strategies in novel tasks, and d) understand their own thinking and learning strategies. (Reflective Thinking: RT)

Teaching students to think reflectively:

B. Provide a general text framework:

1. Two types of study strategies are beneficial for learning from math textbooks:
    1. For reading the explanatory text, employ a general study strategy: SQ5R (UNCW SQ5R and Forgan and Mangrum, 1981, pp. 228-231)
        1. Survey: Read all headings, subheadings, and the final or summary paragraph. Note words in bold print or italics, and any material set apart in some way (color or box). Reflect on the ideas read and formulate a question about them.
        2. Question: turn headings and major points into questions. The question format provides the reader with the most specific direction of any sentence pattern. Write a question about all headings and subheading.
        3. Read: Read to find the answer to each specific question one at a time. The student may skim, skip, or re-read material. The reading style should vary with the material.
        4. Record: The student takes notes on what is read, using his/her own words rather than merely copying from the text. A brief summary of each section of is also written. These notes should be combined with notes from class meetings and other assignments. Vocabulary terms, concepts, and major formulas need to be written in a two-column format for proficient study.
        5. Recite: After an answer has been found, the reader pauses to recite the answer to the question. This helps the student check the clarity of the answer and fix the ideas into memory. Most students should recite aloud or sub vocally if necessary, since most people are less accepting of their own ideas when spoken aloud. Vocabulary should be studied with one column covered while the student recites. Students should alternate columns to establish stronger neural pathways.
        6. Review: Immediately upon completion of the reading assignment, the first review should take place in order to fix the overall organization and the specific ideas into memory. The next should occur within the next 24 hours. Periodic, spaced, reviews should follow to aid in retention.
        7. Reflect: The student reflects on the questions and answers after each review. The goal is to connect with the material on a personal level, integrate the new information with previously learned material, strengthen neural pathways, notice patterns and their relationship to previous patterns and concepts, and develop an understanding of how the new may be used to extend the students abilities within and outside the subject material. Include any important reflections or connection in the written notes. Students should make a list of any reservations, concerns or opinions about the material for clarification during the next class.
    2. A for reading example items and working end of section exercises, use a <u>math specific strategy</u>, especially useful for word problems: SQRQCQR (Forgan and Mangrum, 1981, pp. 238-240)
        1. Survey: Read the complete problem rapidly but carefully to determine the intent or outcome.
        2. Question: Once a clear understanding of the intent is established, write it in question form.

3. Read: Read the problem carefully to determine the pertinent facts.
4. Question: Ask, "What mathematical process must be followed to obtain the correct answer to the question?"
5. Compute: Follow the process through to find an answer.
6. Question: Ask, "Does the answer appear correct?" Make a simple check to verify.
7. Reflect: How does the new fit with the old? Is there a pattern similar to one already studied? What process from previous material is necessary to help solve the new material? (See RoT # 10).

C. Teach a problem solving method or approach:

Polya (1957), in ***How to Solve It***, presents a four-phase problem–solving method. Students who learn the first three stages develop an improved ability to work through items of the type they study, but may experience difficulty in transferring their skill to other problem types. The key to transferring their skill lies in the correct use of step 4 of Polya's Method. Polya called it Looking Back, sometimes referred to as Check, but reading and study skill instructors would recognize it as reflection. Reflection, properly done, helps students develop a deeper understanding of not only the topic studied, but of how they learn and construct their own learning techniques. A mathematics specific study strategy coupled with Polya's method provides students and expanded capability not only as problem solvers, but also as learners.

The following is a listing of Polya's classic "Four phases of the problem-solving process" (Johnson):

I. Understanding the Problem
   (a) Can you state the problem in your own words?
   (b) What are you trying to find or do?
   (c) What are the unknowns?
   (d) What information do you obtain from the problem?
   (e) What information, if any, is missing or not needed?

II. Devising a Plan
   The following list of strategies, although not exhaustive, is very useful:
   (a) Look for a pattern.
   (b) Examine related problems and determine if the same technique can be applied.
   (c) Examine a simpler or special case of the problem to gain insight into the solution of the original problem.
   (d) Make a table.
   (e) Make a diagram.
   (f) Write an equation.
   (g) Use guess and check.
   (h) Work backward.
   (i) Identify a subgoal.

III. Carrying Out the Plan
   (a) Implement the strategy or strategies in step 2 and perform any necessary actions or computations.

(b) Check each step of the plan as you proceed. This may be intuitive checking or a formal proof of each step.
(c) Keep an accurate record of your work.

IV. Looking Back

(a) Check the results in the original problem. (In some cases, this will require a proof)
(b) Interpret the solution in terms of the original problem. Does your answer make sense? Is it reasonable?
(c) Determine whether there is another method of finding the solution.
(d) If possible, determine other related or more general problems for which the techniques will work.

D. Model, require, and assess, either formally or informally, all study behaviors.

Provide examples of reflective thinking while solving a problem. Emphasize process.

Require students to demonstrate reflective thinking during subsequent in–class exercises.

Assess through solo and group presentations, written quizzes and tests, one-on-one discussions of items.

Make clear the connections and patterns within the section, chapter, and from previous material.

Assign vocabulary study, demonstrate a format, and test understanding through proper use in-class and on quizzes and tests. Include non-mathematical vocabulary when students show need.

E. Encourage student engagement (expand math and reflective thinking beyond the classroom).

Provide one or more general discussion starters to begin each class. I use The Rules of Thumb (RoT). The RoT's are an increasing and growing set of sayings developed, stolen and borrowed by the presenter, with a couple generated by his students, based a number of general and mathematical experiences. As the Rule of Thumb is introduced, a brief discussion of how and why it came to be is included and students are asked how it could be helpful outside of class. Often, a situation will arise during a discussion or problem solving session that elicits, "There's a Rule of Thumb about that."

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HOW TO SOLVE IT

UNDERSTANDING THE PROBLEM

First.
You have to *understand* the problem.

What is the unknown? What are the data? What is the condition? Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?

Draw a figure. Introduce suitable notation.

Separate the various parts of the condition. Can you write them down?

DEVISING A PLAN

Second.
Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.

Have you seen it before? Or have you seen the same problem in a slightly different form?

Do you know a related problem? Do you know a theorem that could be useful?

Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.

Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?

Could you restate the problem? Could you restate it still differently? Go back to definitions.

If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or the data, or both if necessary, so that the new unknown and the new data are nearer to each other?

Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?

CARRYING OUT THE PLAN

Third.
Carry out your plan.

Carrying out your plan of the solution, *check each step.* Can you see clearly that the step is correct? Can you prove that it is correct?

LOOKING BACK

Fourth.
Examine the solution obtained.

Can you *check the result?* Can you check the argument?

Can you derive the result differently? Can you see it at a glance?

Can you use the result, or the method, for some other problem?

Dr. O's Rules of Thumb (in part)

2. College is a full time job when you're attending full time (12 or more hours per semester).
3. Being physically present is NOT the same as being being attentively present.
4. Whatever course you're taking or job you have, the most important thing you can do is learn the vocabulary earlier rather than later.
5. Don't cause others to wait on you by being late to class or meetings; don't step out and back–in during the meeting; don't leave before the ending time. You DO miss something important.
6. Finish with your cell phone BEFORE you enter the classroom.
7. Anything you turn in for credit needs to have your name on it. If not, it gets thrown away. Why?
8. If it's shown on the screen, repeated, or given in a handout, it's important and you'll see it again.
9. You learn the best way for you to accomplish a task by: first, accomplish the task; then, second, reflect on the process you used: ask questions about how you did it, what was effective, what was a dead end, and how you could improve.
10. Always have a backup plan.
11. Get in the habit of doing things the way they are meant to be done.
12. If it helps, do it; if it doesn't help, don't do it.
13. When you mess up, fess up, fix it, and learn from it.
14. When you do it right it works better, so you tend to have better results.
15. If something is worth doing, is it not worth doing well?
16. The better you prepare, the easier it is to perform well.
17. You are known by what you do; so, when you put your name on something, what are you telling people?
18. Think first, think again, then speak.
19. Your actions show what is important to you.
20. When your actions show that something is not important to you, why should anyone else think that it is important?
21. Little things matter, such as being on time, filling out forms correctly, having your work ready, saying "please" and "thank you."
22. When you share with others (friends, family, colleagues, classmates), the results are always positive: the task is lessened and the rewards are multiplied.
23. When you let selfishness rule your thoughts and actions, the results are always negative.
24. Steps are never skipped, whether in tasks or on stairs. The same work must still be done.
25. Well organized notes and well organized thoughts lead to better retention and understanding.
26. Life is a never–ending series of applications (math speak for word problems, work speak for opportunities). Become proficient at problems solving.
27. When solving a problem, know where to start AND where to stop.
28. Read the directions FIRST, then follow them.

29. Different does not mean wrong.
30. When you know what to do and how to do it, then, JUST DO IT.
31. Answer the question that is asked. Do NOT assume.
32. Be sure that you understand what question is being asked.
33. As a habit of success, do what is actually meant.
34. The more you know about how to perform a task, the easier it is to get the correct result.
35. On a multiple-choice test with four answer choices, don't pick E.
36. Things are only as hard as they are. Don't make them harder or easier than they have to be.
37. When you come back after missing a class, don't even try to ask the professor if you missed something important! ALWAYS consult your Class Buddies first.
38. Read problems all the way through before you ask questions; listen, think, and filter before speaking.
39. It is easier to take a grade down than to bring a grade up.
40. It is easier to lower a GPA than to bring it up.
41. We learn better inductively, and we use what we've learned better deductively.
42. Concentrate on what you can control. DO NOT waste time on what you cannot control.
43. Your ears work better when your eyes are open and your mouth is closed.
44. To compete successfully, you MUST cooperate successfully.
45. Do NOT put words in Dr. O's mouth (or anyone else's).
46. We are slaves to our habits; therefore, we need to make sure we have good habits.
47. We eat a watermelon by cutting it into pieces, then eating a piece one bite at a time. We solve big problems the same way.
48. Thinking about a task before you begin, while you are working, and after you have completed it is an excellent habit when you learn from it and act upon what you learn.
49. Come to class with questions, leave with answers. ALERT: Answers generally lead to more questions.